MEASURING RELIABILITY OF n-CASCADE SYSTEM UNDER RANDOM STRESS ATTENTION

By

DIGVIJAY SINGH

Deptt. Of Mathematics, J.P Institute of Engg. & Tech., Meerut (INDIA)
drdigvijay2008@rediffmail.com

ABSTRACT: In this paper the author has considered the system reliability of n-cascade system with stress following normal distribution and strength following exponential distribution. They concluded that even for fewer values of stress and strength parameter a high reliability could be high degree of reliability.

KEYWORD: Cascade system, survival function, standby system.

INTRODUCTION

The author has considered the system reliability of n-cascade system with stress following normal distribution and strength following exponential distribution. They concluded that even for fewer values of stress and strength parameter a high reliability could be high degree of reliability. The stress attenuation cascade reliability for a system when both stress and strength are subjected to Rayleigh distribution. They concluded that for lower attenuation factors a high degree of reliability could be attained even when the components are characterized with linearly increasing failure rate.

MATHEMATICAL MODEL

The n-cascade system, a special type of standby system with n components, was defined by Shrivastav and Pandit [1]. Cascade redundancy is such a standby redundancy, where a standby component takes the place of the failed component with changed stress. This changed stress is k times the preceding stress and it is called the attenuation factor. Consider a system with n components $C_1, C_2, C_3, \ldots, C_n$ with strengths $x_1, x_2, x_3, \ldots, x_n$ arranged in order of activation. Let $f(x_i),\ i = 1, 2, 3, \ldots, n$ be the probability density functions for the independently distributed random variables $x_1, x_2, x_3, \ldots, x_n$. Also, let $g(y_1)$ be the probability density function for the randomly distributed stress on the first component y_1. The stress on the components undergoes attenuation at each failure by a random factor k. Their stress attenuation factors $k_1, k_2, k_3, \ldots, k_n$ are defined over intervals $l_1, l_2, l_3, \ldots, l_n$ with probability functions $h(k_i),\ i = 1, 2, 3, \ldots, n$. In n-cascade system, if the stress exceeds the strength, then it leads to the failure of the components and the next component in the sequence gets activated. Hence, the system reliability R(n) for the nth component can be evaluated only when the first (n – 1) components fail and the nth component activates and is given by

$$R(n) = P\left[\left\{\bigcap_{i=1}^{n-1} (x_i < y_i)\right\} \cap (x_n > y_n)\right].$$

Thus for n = 1, 2, 3 and 4 the reliability expression are

(a) $$R(1) = P[x_1 > y_1]$$

$$= \int_0^\infty g(y_1)\,dy_1 \int_{y_1}^\infty f(x_1)\,dx_1,$$

(b)

$$R(2) = P[(x_1 < y_1) \cap (x_2 > y_2)],$$

$$= \int_0^\infty g(y_1)\,dy_1 \int_0^{y_1} f(x_1)\,dx_1 \int_0^1 h(k_2)\,dk_2 \int_{k_2 y_1}^\infty f(x_2)\,dx_2$$

(c)

$$R(3) = P[(x_1 < y_1) \cap (x_2 < y_2) \cap (x_3 > y_3)]$$

$$= \int_0^\infty g(y_1)\,dy_1 \int_0^{y_1} f(x_1)\,dx_1 \int_0^1 h(k_2)\,dk_2 \int_0^{k_2 y_1} f(x_2)\,dx_2$$

$$\int_0^1 h(k_3)\,dk_3 \int_{k_3 k_2 y_1}^\infty f(x_3)\,dx_3,$$

(d)

$$R(4) = P[(x_1 < y_1) \cap (x_2 < y_2) \cap (x_3 < y_3) \cap (x_4 > y_4)]$$

$$= \int_0^\infty g(y_1)\,dy_1 \int_0^{y_1} f(x_1)\,dx_1 \int_0^1 h(k_2)\,dk_2 \int_0^{k_2 y_1} f(x_2)\,dx_2$$

$$\int_0^1 h(k_3)\,dk_3 \int_0^{k_3 k_2 y_1} f(x_3)\,dx_3 \int_0^1 h(k_4)\,dk_4 \int_{k_4 k_3 k_2 y_1}^\infty f(x_4)\,dx_4$$

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EXPONENTIAL STRESS, STRENGTH AND BETA ATTENUATION FACTORS

When stress and strength follow exponential distribution with the probability density functions

(a)

$$f(x_i) = \lambda_i e^{-\lambda_i x_i}, \quad i = 1, 2, 3, ..., n,$$

(b) $$g(y_1) = \mu e^{-\mu y_1}.$$

$$h_{(k_i)} = \frac{k_i^{a-1}(1-k_i)^{b-1}}{\beta(a,b)}, \quad i = 1, 2, 3, ..., n, \quad a, b > 0$$

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Then the expressions for reliability are

(a) $$R(1) = \frac{\mu}{\mu + \lambda_1},$$

(b)

$$R(2) = \frac{\mu}{\beta(a,b)} \sum_{r=0}^{\infty} \left[\frac{(-1)^r \lambda_2^r \beta(r+a,b)}{\lfloor r} \lceil (r+1) \left(\frac{1}{\mu^{r+1}} - \frac{1}{(\mu+\lambda_1)^{r+1}} \right) \right]$$

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$$R(3) = \frac{\mu}{\{\beta(a,b)\}^2} \sum_{r=1}^{\infty} \sum_{s=0}^{\infty} \left[\frac{(-1)^{r+s} \lambda_2^r \lambda_3^s \beta(r+s+a,b) \beta(s+a,b)}{\lfloor r \lfloor s} \right.$$

$$\left. \lceil (r+s+1) \left\{ \frac{1}{\mu^{r+s+1}} - \frac{1}{(\mu+\lambda_1)^{r+s+1}} \right\} \right],$$

(d)

$$R(4) = \frac{\mu}{\{\beta(a,b)\}^3} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \sum_{t=0}^{\infty} \left[\frac{(-1)^{r+s+t} \lambda_2^r \lambda_3^s \lambda_4^t}{\lfloor r \lfloor s \lfloor t} \beta(s+t+a,b) \right.$$

$$\left. \beta(t+a,b)\, \beta(r+s+t+a,b) \lceil (r+s+t+1) \left\{ \frac{1}{\mu^{r+s+t+1}} - \frac{1}{(\mu+\lambda_1)^{r+s+t+1}} \right\} \right].$$

The reliability of the system is given by

$$\tilde{R} = R_1(1-R_2)(1-R_3)(1-R_4)$$

The expressions for the survival function as follows

$$q(1) = \frac{\lambda}{\lambda + \mu_1},$$

$$q(2) = \frac{\lambda}{\beta(a,b)} \sum_{r=0}^{\infty} \left[\frac{(-1)^r \mu_2^r \beta(r+a,b)}{\lfloor r} \lceil (r+1) \left(\frac{1}{\lambda^{r+1}} - \frac{1}{(\lambda+\mu_1)^{r+1}} \right) \right]$$

,

$$q(3) = \frac{\lambda}{\{\beta(a,b)\}^2} \sum_{r=0}^{\infty} \sum_{s=1}^{\infty} \left[\frac{(-1)^{r+s+1} \mu_2^r \mu_3^s \beta(r+s+a,b) \beta(r+a,b)}{\lfloor r \lfloor s} \right.$$

$$\left. \lceil (r+s+1) \left\{ \frac{1}{\lambda^{r+s+1}} - \frac{1}{(\lambda+\mu_1)^{r+s+1}} \right\} \right],$$

$$q(4) = \frac{\lambda}{\{\beta(a,b)\}^3} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \sum_{t=1}^{\infty} \left[\frac{(-1)^{r+s+t+2} \mu_2^r \mu_3^s \mu_4^t}{\lfloor r \lfloor s} \beta(s+t+a,b) \right.$$

$$\left. \beta(t+a,b)\, \beta(r+s+t+a,b) \lceil (r+s+t+1) \left\{ \frac{1}{\lambda^{r+s+t+1}} - \frac{1}{(\lambda+\mu_1)^{r+s+t+1}} \right\} \right].$$

The survival of the system is given by

$$\tilde{q} = q_1(1-q_2)(1-q_3)(1-q_4)$$

Table 1 : Reliability for $\mu = 1$ **and** $\lambda_i = i\lambda_1$

μ	R(1)	R(2)	R(3)	R(4)	$\tilde{R}$
0.01	0.9901	0.0097	-0.0002	0.0000	0.9807
0.02	0.9804	0.0189	-0.0007	0.0000	0.9625
0.03	0.9709	0.0275	-0.0015	0.0001	0.9455
0.04	0.9615	0.0357	-0.0025	0.0002	0.9293
0.05	0.9524	0.0434	-0.0037	0.0004	0.9140
0.06	0.9434	0.0507	-0.0051	0.0007	0.8995
0.07	0.9346	0.0576	-0.0066	0.0010	0.8857
0.08	0.9259	0.0642	-0.0081	0.0013	0.8723
0.09	0.9174	0.0704	-0.0098	0.0017	0.8597
0.10	0.9091	0.0763	-0.0115	0.0021	0.8476

Table 2 : Reliability for $\mu = 5$ and $\lambda_i = i\lambda_1$

μ	R(1)	R(2)	R(3)	R(4)	$\tilde{R}$
0.01	0.9980	0.0020	0.0000	0.0000	0.9960
0.02	0.9960	0.0040	0.0000	0.0000	0.9920
0.03	0.9940	0.0059	-0.0001	0.0000	0.9882
0.04	0.9921	0.0078	-0.0001	0.0000	0.9845
0.05	0.9901	0.0097	-0.0002	0.0000	0.9807
0.06	0.9881	0.0116	-0.0003	0.0000	0.9769
0.07	0.9862	0.0134	-0.0004	0.0000	0.9734
0.08	0.9843	0.0153	-0.0005	0.0000	0.9697
0.09	0.9823	0.0171	-0.0006	0.0000	0.9661
0.10	0.9804	0.0189	-0.0007	0.0000	0.9625

Table 5 : Survival function for $\lambda = 1$ and $\mu_i = i\,\mu_1$

μ	q(1)	q(2)	q(3)	q(4)	$\tilde{q}$
0.01	0.9901	0.0097	0.0001	0.0000	0.9804
0.02	0.9804	0.0189	0.0005	0.0000	0.9614
0.03	0.9709	0.0275	0.0011	0.0001	0.9431
0.04	0.9615	0.0357	0.0018	0.0004	0.9251
0.05	0.9524	0.0434	0.0027	0.0011	0.9076
0.06	0.9434	0.0507	0.0036	0.0023	0.8903
0.07	0.9346	0.0576	0.0046	0.0041	0.8731
0.08	0.9259	0.0642	0.0057	0.0069	0.8556
0.09	0.9174	0.0704	0.0068	0.0106	0.8380
0.10	0.9091	0.0763	0.0079	0.0154	0.8203

Table 3 : Reliability for $\mu = 20$ and $\lambda_i = i\lambda_1$

μ	R(1)	R(2)	R(3)	R(4)	$\tilde{R}$
0.01	0.9995	0.0005	0.0000	0.0000	0.9990
0.02	0.9990	0.0010	0.0000	0.0000	0.9980
0.03	0.9985	0.0015	0.0000	0.0000	0.9970
0.04	0.9980	0.0020	0.0000	0.0000	0.9960
0.05	0.9975	0.0025	0.0000	0.0000	0.9950
0.06	0.9970	0.0030	0.0000	0.0000	0.9940
0.07	0.9965	0.0035	0.0000	0.0000	0.9930
0.08	0.9960	0.0040	0.0000	0.0000	0.9920
0.09	0.9955	0.0044	0.0000	0.0000	0.9911
0.10	0.9950	0.0049	0.0000	0.0000	0.9901

Table 6 : Survival function for $\lambda = 5$ and $\mu_i = i\,\mu_1$

μ	q(1)	q(2)	q(3)	q(4)	$\tilde{q}$
0.01	0.9980	0.0020	0.0000	0.0000	0.9960
0.02	0.9960	0.0040	0.0000	0.0000	0.9920
0.03	0.9940	0.0059	0.0001	0.0000	0.9880
0.04	0.9921	0.0078	0.0001	0.0000	0.9843
0.05	0.9901	0.0097	0.0001	0.0000	0.9804
0.06	0.9881	0.0116	0.0002	0.0000	0.9764
0.07	0.9862	0.0134	0.0003	0.0000	0.9727
0.08	0.9843	0.0153	0.0003	0.0000	0.9689
0.09	0.9823	0.0171	0.0004	0.0000	0.9651
0.10	0.9804	0.0189	0.0005	0.0000	0.9614

Table 4 : Reliability for $\mu = 25$ and $\lambda_i = i\lambda_1$

μ	R(1)	R(2)	R(3)	R(4)	$\tilde{R}$
0.01	0.9996	0.0004	0.0000	0.0000	0.9992
0.02	0.9992	0.0008	0.0000	0.0000	0.9984
0.03	0.9988	0.0012	0.0000	0.0000	0.9976
0.04	0.9984	0.0016	0.0000	0.0000	0.9968
0.05	0.9980	0.0020	0.0000	0.0000	0.9960
0.06	0.9976	0.0024	0.0000	0.0000	0.9952
0.07	0.9972	0.0028	0.0000	0.0000	0.9944
0.08	0.9968	0.0032	0.0000	0.0000	0.9936
0.09	0.9964	0.0036	0.0000	0.0000	0.9928
0.10	0.9960	0.0040	0.0000	0.0000	0.9920

Table 7 : Survival function for $\lambda = 20$ and $\mu_i = i\,\mu_1$

μ	q(1)	q(2)	q(3)	q(4)	$\tilde{q}$
0.01	0.9995	0.0005	0.0000	0.0000	0.9990
0.02	0.9990	0.0010	0.0000	0.0000	0.9980
0.03	0.9985	0.0015	0.0000	0.0000	0.9970
0.04	0.9980	0.0020	0.0000	0.0000	0.9960
0.05	0.9975	0.0025	0.0000	0.0000	0.9950
0.06	0.9970	0.0030	0.0000	0.0000	0.9940
0.07	0.9965	0.0035	0.0000	0.0000	0.9930
0.08	0.9960	0.0040	0.0000	0.0000	0.9920
0.09	0.9955	0.0044	0.0000	0.0000	0.9911
0.10	0.9950	0.0049	0.0000	0.0000	0.9901

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Table 8 : Survival function for $\lambda = 25$ **and** $\mu_i = i\,\mu_1$

μ	q(1)	q(2)	q(3)	q(4)	$\tilde{q}$
0.01	0.9996	0.0004	0.0000	0.0000	0.9992
0.02	0.9992	0.0008	0.0000	0.0000	0.9984
0.03	0.9988	0.0012	0.0000	0.0000	0.9976
0.04	0.9984	0.0016	0.0000	0.0000	0.9968
0.05	0.9980	0.0020	0.0000	0.0000	0.9960
0.06	0.9976	0.0024	0.0000	0.0000	0.9952
0.07	0.9972	0.0028	0.0000	0.0000	0.9944
0.08	0.9968	0.0032	0.0000	0.0000	0.9936
0.09	0.9964	0.0036	0.0000	0.0000	0.9928
0.10	0.9960	0.0040	0.0000	0.0000	0.9920

Interpretation of Result

Numerical results are calculated both for reliability (for $\lambda_i = i\lambda_1$ and λ_1 ranging from 0.01 to 0.10 in step of 0.01) and the survival function for ($\mu_i = i\mu_1$ and μ_1 ranging from 0.01 to 0.10) for different stress and strength parameter.

From the above results, we conclude that $\tilde{R}$ decreases with the increase in λ_1 but increases due to increase in values of parameter μ .

Similarly, $\tilde{q}$ also increases with the increase of value of λ_1 but decreases as μ_1 range from 0.01 to 0.10.

REFERENCES

[1] Srivastav G. L. and Pandit S. N. (1975) : Studies on cascade reliability – I, IEEE Trans. Reliability, 53-60.

[2] Raghava Char, A. C. N., KeshavRao B. and PanditS. N. N. (1988) : Survival functions of a component under strength attenuation, IAPQR Trans.,13.

[3] Evans J. R. and Lindsay W. M. (2000) : The management and control of quality, 5th Ed., South Western Publishing, Cincinnati, OH.

[4] Gunes M. and Deveci I. (2002) : Reliability of service systems and an application in student office, International Journal of Quality and Reliability, Vol. 19, 206-14.